

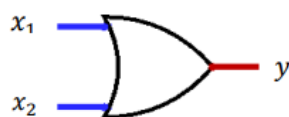
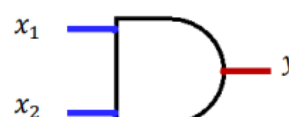
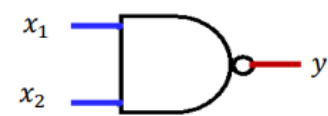


Digitalna vezja UL, FRI



Vaja 2 Funkcije, PDNO in PKNO

Osnovne preklopne funkcije

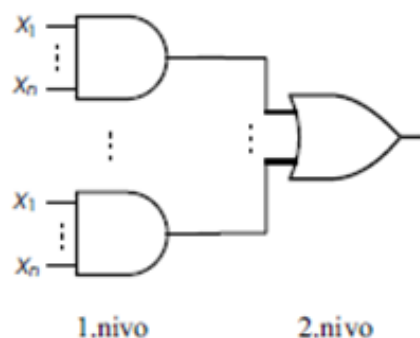
Negacija (NE , NOT)	Disjunkcija (ALI, OR)	Konjunkcija (IN, AND)	Percipov operator (NE ALI, NOR)																																																												
$y = \bar{x}$	$y = x_1 \vee x_2$	$y = x_1 \cdot x_2$	$y = x_1 \downarrow x_2 = \overline{x_1 \vee x_2}$																																																												
																																																															
<table border="1"><thead><tr><th>x</th><th>$y = \bar{x}$</th></tr></thead><tbody><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></tbody></table>	x	$y = \bar{x}$	0	1	1	0	<table border="1"><thead><tr><th>x_1</th><th>x_2</th><th>$y = x_1 \vee x_2$</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></tbody></table>	x_1	x_2	$y = x_1 \vee x_2$	0	0	0	0	1	1	1	0	1	1	1	1	<table border="1"><thead><tr><th>x_1</th><th>x_2</th><th>$y = x_1 \cdot x_2$</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></tbody></table>	x_1	x_2	$y = x_1 \cdot x_2$	0	0	0	0	1	0	1	0	0	1	1	1	<table border="1"><thead><tr><th>x_1</th><th>x_2</th><th>$y = x_1 \downarrow x_2$</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></tbody></table>	x_1	x_2	$y = x_1 \downarrow x_2$	0	0	1	0	1	0	1	0	0	1	1	0									
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Shefferjev operator (NE IN, NAND)	Ekskluzivni ALI (XOR)	Ekvivalenca (XNOR)	Implikacija																																																												
$y = x \uparrow y = \overline{x \cdot y}$	$y = x_1 \nabla x_2 = \overline{x_1} \cdot x_2 \vee x_1 \cdot \overline{x_2}$	$y = x_1 \equiv x_2 = \overline{x_1} \cdot \overline{x_2} \vee x_1 \cdot x_2$	$y = x_1 \rightarrow x_2 = \overline{x_1} \vee x_2$																																																												
																																																															
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Zapis popolnih oblik preklopnih funkcij

Normalna oblika – dva nivoja operatorjev; na posameznem nivoju le en operator

Popolna oblika – v vseh izrazih prvega nivoja so vse spremenljivke

Popolna disjunktivna normalna oblika (PDNO)



$$f(x_1, x_2, \dots, x_n) = \bigvee_{i=0}^{2^n-1} m_i f(\vec{w}_i)$$

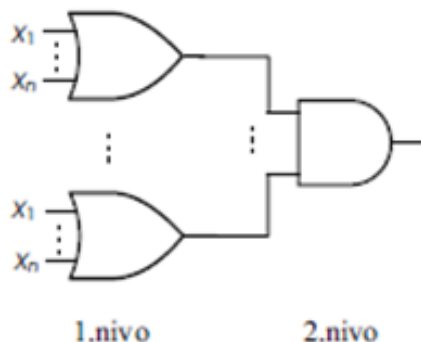
$f(\vec{w}_i)$... vrednost funkcije pri i -tem vhodnem vektorju (vrstici)

m_i ... minterm i ; $m_i = x_1^{w_{1,i}} \cdot x_2^{w_{2,i}} \cdot \dots \cdot x_n^{w_{n,i}}$; $i = 0, 1, 2, \dots, 2^n - 1$

$$x^w = \begin{cases} x, & w = 1 \\ \bar{x}, & w = 0 \end{cases}$$

$w_{j,i} \dots j$ -ti bit binarnega zapisa števila i

Popolna konjunktivna normalna oblika (PKNO)



$$f(x_1, x_2, \dots, x_n) = \bigwedge_{i=0}^{2^n-1} (M_i \vee f(\vec{w}_i))$$

$f(\vec{w}_i)$... vrednost funkcije pri i -tem vhodnem vektorju (vrstici)

M_i ... maksterm i ; $M_i = \bar{x}_1^{w_{1,i}} \vee \bar{x}_2^{w_{2,i}} \vee \dots \vee \bar{x}_n^{w_{n,i}}$; $i = 0, 1, 2, \dots, 2^n - 1$

Primer: Preklopna funkcija $f(x_1, x_2, x_3)$

Minterm – konjunkcija vhodnih spremenljivk x_1, x_2, x_3 . Spremenljivka x_i je negirana, če je vhodna vrednost enaka 0, in je nenegirana, če je vhodna vrednost enaka 1.

Maksterm - disjunkcija vhodnih spremenljivk x_1, x_2, x_3 . Spremenljivka x_i je negirana, če je vhodna vrednost enaka 1 in je nenegirana, če je vhodna vrednost enaka 0.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	minterm	maksterm
0	0	0	$f(\overrightarrow{w_0})$	$m_0 = \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}$	$M_0 = x_1 \vee x_2 \vee x_3$
0	0	1	$f(\overrightarrow{w_1})$	$m_1 = \overline{x_1} \cdot \overline{x_2} \cdot x_3$	$M_1 = x_1 \vee x_2 \vee \overline{x_3}$
0	1	0	$f(\overrightarrow{w_2})$	$m_2 = \overline{x_1} \cdot x_2 \cdot \overline{x_3}$	$M_2 = x_1 \vee \overline{x_2} \vee x_3$
0	1	1	$f(\overrightarrow{w_3})$	$m_3 = \overline{x_1} \cdot x_2 \cdot x_3$	$M_3 = x_1 \vee \overline{x_2} \vee \overline{x_3}$
1	0	0	$f(\overrightarrow{w_4})$	$m_4 = x_1 \cdot \overline{x_2} \cdot \overline{x_3}$	$M_4 = \overline{x_1} \vee x_2 \vee x_3$
1	0	1	$f(\overrightarrow{w_5})$	$m_5 = x_1 \cdot \overline{x_2} \cdot x_3$	$M_5 = \overline{x_1} \vee x_2 \vee \overline{x_3}$
1	1	0	$f(\overrightarrow{w_6})$	$m_6 = x_1 \cdot x_2 \cdot \overline{x_3}$	$M_6 = \overline{x_1} \vee \overline{x_2} \vee x_3$
1	1	1	$f(\overrightarrow{w_7})$	$m_7 = x_1 \cdot x_2 \cdot x_3$	$M_7 = \overline{x_1} \vee \overline{x_2} \vee \overline{x_3}$

PDNO

Zapišite PDNO za preklopno funkcijo $f(x_1, x_2, x_3) = x_1 \cdot \overline{x_2} \vee x_2 \cdot x_3$.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	minterm
0	0	0	0	$m_0 = \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}$
0	0	1	0	$m_1 = \overline{x_1} \cdot \overline{x_2} \cdot x_3$
0	1	0	0	$m_2 = \overline{x_1} \cdot x_2 \cdot \overline{x_3}$
0	1	1	1	$m_3 = \overline{x_1} \cdot x_2 \cdot x_3$
1	0	0	1	$m_4 = x_1 \cdot \overline{x_2} \cdot \overline{x_3}$
1	0	1	1	$m_5 = x_1 \cdot \overline{x_2} \cdot x_3$
1	1	0	0	$m_6 = x_1 \cdot x_2 \cdot \overline{x_3}$
1	1	1	1	$m_7 = x_1 \cdot x_2 \cdot x_3$

$$\begin{aligned} f(x_1, x_2, x_3) &= m_0 \cdot 0 \vee m_1 \cdot 0 \vee m_2 \cdot 0 \vee m_3 \cdot 1 \vee m_4 \cdot 1 \vee m_5 \cdot 1 \vee m_6 \cdot 0 \vee m_7 \cdot 1 \\ &= m_3 \vee m_4 \vee m_5 \vee m_7 = \\ &= \vee^3(3, 4, 5, 7) = \\ &= \overline{x_1} \cdot x_2 \cdot x_3 \vee x_1 \cdot \overline{x_2} \cdot \overline{x_3} \vee x_1 \cdot \overline{x_2} \cdot x_3 \vee x_1 \cdot x_2 \cdot x_3 \end{aligned}$$

PKNO

Zapišite PKNO za preklapno funkcijo $f(x_1, x_2, x_3) = x_1 \cdot \overline{x_2} \vee x_2 \cdot x_3$.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	maksterm
0	0	0	0	$M_0 = x_1 \vee x_2 \vee x_3$
0	0	1	0	$M_1 = x_1 \vee x_2 \vee \overline{x_3}$
0	1	0	0	$M_2 = x_1 \vee \overline{x_2} \vee x_3$
0	1	1	1	$M_3 = x_1 \vee \overline{x_2} \vee \overline{x_3}$
1	0	0	1	$M_4 = \overline{x_1} \vee x_2 \vee x_3$
1	0	1	1	$M_5 = \overline{x_1} \vee x_2 \vee \overline{x_3}$
1	1	0	0	$M_6 = \overline{x_1} \vee \overline{x_2} \vee x_3$
1	1	1	1	$M_7 = \overline{x_1} \vee \overline{x_2} \vee \overline{x_3}$

$$\begin{aligned} f(x_1, x_2, x_3) &= (M_0 \vee 0) \cdot (M_1 \vee 0) \cdot (M_2 \vee 0) \cdot (M_3 \vee 1) \cdot (M_4 \vee 1) \cdot (M_5 \vee 1) \cdot (M_6 \vee 0) \cdot (M_7 \vee 1) \\ &= M_0 \cdot M_1 \cdot M_2 \cdot M_6 = \\ &= \&^3(0, 1, 2, 6) = \\ &= (x_1 \vee x_2 \vee x_3) \cdot (x_1 \vee x_2 \vee \overline{x_3}) \cdot (x_1 \vee \overline{x_2} \vee x_3) \cdot (\overline{x_1} \vee \overline{x_2} \vee x_3) \end{aligned}$$

Pretvorba PDNO $\leftrightarrow$ PKNO

Relacije med mintermi in makstermi:

$$\begin{array}{ll} \overline{m_i} = M_i & m_i \vee M_i = 1 \\ \overline{M_i} = m_i & m_i \cdot M_i = 0 \end{array}$$

Pretvorba PDNO $\leftrightarrow$ PKNO:

1. Funkcijo negiramo (izpišemo komplementarne terme).
2. Manjkajoči indeksi predstavljajo indekse dualnih termov.

$$f(x_1, x_2, x_3) = x_1 \cdot \overline{x_2} \vee x_2 \cdot x_3$$

PDNO: $\vee^3 (3, 4, 5, 7)$

$\leftrightarrow$

PKNO: $\&^3 (0, 1, 2, 6)$

Naloga

Preklopni funkciji $f(x_1, x_2, x_3) = x_1 \cdot \overline{x_2} \vee \overline{x_3}$ določite pravilnostno tabelo ter zapišite PDNO. Z uporabo relacij med mintermi in makstermi pretvorite PDNO v PKNO in jo zapišite v skrajšani in eksplicitni obliki. Dobljen PKNO nato narišite v Logisimu.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	minterm	maksterm
0	0	0		$m_0 = \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}$	$M_0 = x_1 \vee x_2 \vee x_3$
0	0	1		$m_1 = \overline{x_1} \cdot \overline{x_2} \cdot x_3$	$M_1 = x_1 \vee x_2 \vee \overline{x_3}$
0	1	0		$m_2 = \overline{x_1} \cdot x_2 \cdot \overline{x_3}$	$M_2 = x_1 \vee \overline{x_2} \vee x_3$
0	1	1		$m_3 = \overline{x_1} \cdot x_2 \cdot x_3$	$M_3 = x_1 \vee \overline{x_2} \vee \overline{x_3}$
1	0	0		$m_4 = x_1 \cdot \overline{x_2} \cdot \overline{x_3}$	$M_4 = \overline{x_1} \vee x_2 \vee x_3$
1	0	1		$m_5 = x_1 \cdot \overline{x_2} \cdot x_3$	$M_5 = \overline{x_1} \vee x_2 \vee \overline{x_3}$
1	1	0		$m_6 = x_1 \cdot x_2 \cdot \overline{x_3}$	$M_6 = \overline{x_1} \vee \overline{x_2} \vee x_3$
1	1	1		$m_7 = x_1 \cdot x_2 \cdot x_3$	$M_7 = \overline{x_1} \vee \overline{x_2} \vee \overline{x_3}$

PDNO: $f(x_1, x_2, x_3) = ?$

PKNO: $f(x_1, x_2, x_3) = ?$